# Programlama -1

# “Matris”

Dr. Cahit Karakuş, 2020

# Vectors

- Yalnızca bir satırı olan bir matrise satır matrisi denir.
- Yalnızca bir sütunu olan bir matrise sütun matrisi denir.
- Satır matrisine satır vektörü, sütun matrisine sütun vektörü denir.

# Scalars

A 1 x 1 matrix is called a *scalar* and is the form of most variables considered in simple algebraic forms.

# Matrices

- Matrisler, sistemlerin analizini sistematik bir şekilde geliştirmek için değerlerin veya fonksiyonların düzenli bir şekilde düzenlenmesini sağlar.
- Geniş lineer denklem dizilerini basitleştirmede ve çözümlerini belirlemede kullanımları çok önemlidir.
- Özellikle, büyük dizileri işlemek için bilgisayar algoritmaları, matris gösterimi ve işlemleri ile büyük ölçüde basitleştirilmiştir.
- Bu noktada matrisleri tanıtmanın özel bir nedeni, MATLAB'ın ağırlıklı olarak matris yönelimli olmasıdır.
- MATLAB'ın güçlü özelliklerinden yararlanmak için matris teorisinde uzman olmak gerekmez, ancak bazı terminoloji ve manipülasyonları (amaca yönelik işlem) anlamak çok yararlıdır.
- Lineer cebir terimi genellikle matrislerin genel teorisini ve ilişkili cebirsel işlemleri temsil etmek için kullanılır.

# Arrays and Matrices

```
v = [-2 3 0 4.5 -1.5];  % length 5 row vector.
v = v’;                 % transposes v.
v(1);                   % first element of v.
v(2:4);                 % entries 2-4 of v.
v([3,5]);               % returns entries 3 & 5.
v=[4:-1:2];             % same as v=[4 3 2];
a=1:3; b=2:3; c=[a b]; → c = [1 2 3 2 3];
```

# Skaler ve Vektörel Çarpım

```
a=3
b=9
c1=a+b
c2=a-b
c3=a/b
c4=a*b
V1=[1 3 -1 2]
V2=[2;-1;2;0]
V7=[2 -1 2 0]
V3=a*V1;
V4=V1*a;

V5=V1*V2;
V6=V1.*V7
```

- a,b,c1,c2,c3,c4 skaler büyüklüklerdir.
- V1,V2, V3, V4, V5 ise vektörel büyüklükler
- V5=V1*V2 vektörel çarpım
- V5=1*2+3*(-1)+(-1)*2+2*0=-3
- Skaler çarpımda herbir vektörün herbir elemanı birbirleriyle çarpılır. Satır vektörü satır vektörü ile sütun vektörü sütun vektörü ile çarpılır. Skaler çarpılacak vektörel büyüklüklein boyutları da eşit olmak zorundadır.

```
V1 =     1     3    -1     2

V7 =     2    -1     2     0
V6 =     2    -3    -2     0
```

# Skaler ve Matris Çarpım

```
a=3
V1=[1 3 -1 2; 2 0 1 -1; 4 3 1 1; 5 5 4 6]
V3=a*V1
```

```
a =     3
V1 =
     1     3    -1     2
     2     0     1    -1
     4     3     1     1
     5     5     4     6

V3 =
     3     9    -3     6
     6     0     3    -3
    12     9     3     3
    15    15    12    18
```

- Skaler ile matris çarpımında matrisin herbir elemanı skaler değer ile tek tek çarpılır.

# Arrays and Matrices (2)

**x = linspace(-pi,pi,10);** % creates 10 linearly-spaced elements from –pi to pi.

**logspace** is similar.

**A = [1 2 3; 4 5 6];** % creates 2x3 matrix

**A(1,2)** % the element in row 1, column 2.

**A(:,2)** % the second column.

**A(2,:)** % the second row.

# Arrays and Matrices (3)

**A+B**, **A-B**, **2*A**, **A*B** % matrix addition, matrix subtraction, scalar multiplication, matrix multiplication

**A.*B** % element-by-element mult.

**A’** % transpose of A (complex- conjugate transpose)

**det(A)** % determinant of A

# Vektör ve Matris  Vektörel Çarpım

```
V1=[1 3 -1 2; 2 0 1 -1; 4 3 1 1; 5 5 4 6]
V2=[2;-1;2;0]
V3=V1*V2
```

- vektör ile matris çarpımında  V4=V2*V1 yazılamaz; vektörel çarpım kuralına uymadı.
- .V3=V1*V2 yazılmalıdır.

```
V1 =
     1     3    -1     2
     2     0     1    -1
     4     3     1     1
     5     5     4     6

V2 =
     2
    -1
     2
     0
```

$$
\begin{array}{rrrr} 1 & 3 & -1 & 2 \\ 2 & 0 & 1 & -1 \\ 4 & 3 & 1 & 1 \\ 5 & 5 & 4 & 6 \end{array}
\qquad
\begin{array}{r} 2 \\ -1 \\ 2 \\ 0 \end{array}
\qquad
\begin{array}{r} -3 \\ 6 \\ 7 \\ 13 \end{array}
$$

# Creating special matrices

| | |
|---|---|
| **diag(v)** | % change a vector v to a diagonal matrix. |
| **diag(A)** | % get diagonal of A. |
| **eye(n)** | % identity matrix of size n. |
| **zeros(m,n)** | % m-by-n zero matrix. |
| **ones(m,n)** | % m*n matrix with all ones. |

# More matrix/vector operations

| | |
|---|---|
| **length(v)** | % determine length of vector. |
| **size(A)** | % determine size of matrix. |
| **rank(A)** | % determine rank of matrix. |
| **norm(A)**, **norm(A,1)**, **norm(A,inf)** | % determine 2-norm, 1-norm,<br>and infinity-norm of A. |
| • **norm(v)** | % compute vector 2-norm. |

# Vektör ve Matris Skaler Çarpım

```
V1=[1 3 -1 2; 2 0 1 -1; 4 3 1 1; 5 5 4 6]
V2=[2;-1;2;0]
V3=V1*V2
```

- vektör ile matris vektörel çarpım olamaz.

```
V1 =
     1     3    -1     2
     2     0     1    -1
     4     3     1     1
     5     5     4     6

V2 =
     2
    -1
     2
     0
```

$$\begin{matrix} 1 & 3 & -1 & 2 \\ 2 & 0 & 1 & -1 \\ 4 & 3 & 1 & 1 \\ 5 & 5 & 4 & 6 \end{matrix} \qquad \begin{matrix} 2 \\ -1 \\ 2 \\ 0 \end{matrix} \qquad \begin{matrix} -3 \\ 6 \\ 7 \\ 13 \end{matrix}$$

# Matris ve Matris Skaler Çarpım

`Z = X.*exp(-((X- Y.^2).^2+Y.^2));`

- X bir matris, Y de bir matris. O halde matrisin matris ile skaler çarpımında X’in saü tarafına bir nokta konur.
- Y.^2 ifadesinde de Y matristir. O hale Y^2 ifadesi matrisin matris ile skaler çarpımıdır. Bu nedenle nokta konur.

# General Form of a Matrix of Size *m* x *n* with *m* Rows and *n* Columns

$$\mathbf{A} = \begin{bmatrix} a_{11} \; a_{12} \; a_{13} \ldots\ldots a_{1n} \\ a_{21} \; a_{22} \; a_{23} \ldots\ldots a_{2n} \\ \vdots \qquad\qquad\qquad\qquad \vdots \\ a_{m1} \; a_{m2} \; a_{m3} \ldots a_{mn} \end{bmatrix}$$

# Square Matrix

If *m* = *n*, the matrix is said to be *square.* In this case, the matrix could be designated as an m x m matrix.

$$\mathbf{A}_{\mathbf{m,m}}$$

# Transpose (Devriği)

Bir A matrisinin devriği A' olarak gösterilir ve satırlar ile sütunların yer değiştirmesiyle elde edilir. Böylece, eğer A'nın boyutu mxn ise, A‵ nın boyutu da nxm olacaktır. Devrik işlemi iki kez uygulanırsa, orijinal matris geri yüklenir.

Example: Determine the size of the matrix shown below.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

The matrix has 2 rows and 3 columns. Its size is 2 x 3.

# Example: Determine the size of the matrix shown below.

$$\mathbf{B} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

The matrix has 3 rows and 2 columns. Its size is 3 x 2.

# Entering a Matrix in MATLAB

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

MATLAB Format

```
A = [2 -3 5; -1 4 5]
A =
     2    -3     5
    -1     4     5
```

# Entering a Row Vector in MATLAB

$$\mathbf{x} = [1\ 4\ 7]$$

MATLAB Format

```
x = [1 4 7]
x =
     1     4     7
```

# Entering a Column Vector in MATLAB

$$\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$$

MATLAB Format

```
x = [1; 4; 7]
x =
     1
     4
     7
```

# Alternate Way to Enter a Column Vector

```
x = [1 4 7]'
x =
    1
    4
    7
```

# Example: Determine the size of the matrix shown below.

$$\mathbf{C} = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 1 \\ -5 & 2 & 1 \end{bmatrix}$$

The matrix has 3 rows and 3 columns. Its size is 3 x 3. It is a *square* matrix.

# Example: Express the integer values of time from 0 to 5 s as a row vector.

$$\mathbf{t} = [0\ 1\ 2\ 3\ 4\ 5]$$

The size of the vector is 1 x 6.

# Example: Express the variables $x_1$, $x_2$, and $x_3$ as a column vector.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Example: Determine the transpose of the matrix **A** below.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix}$$

$$\mathbf{A'} = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 5 & 6 \end{bmatrix}$$

# Addition and Subtraction of Matrices

Matrices can be added together or subtracted from each other if and only if they are of the same size. Corresponding elements are added or subtracted. Toplamda satır ve sütün sayıları birbirlerine eşit olmalıdır.

$$\mathbf{C_{m,n}} = \mathbf{A_{m,n}} \pm \mathbf{B_{m,n}}$$

# Multiplication of Two Matrices

Two matrices can be multiplied together only if the number of columns of the first matrix is equal to the number of rows of the second matrix. This means that

$$\mathbf{AB} \neq \mathbf{BA}$$

# Multiplication of Two Matrices (Continuation)

The number of rows in the product matrix is equal to the number of rows in the first matrix and the number of columns in the product matrix is equal to the number of columns in the second matrix.

$$\mathbf{A}_{\mathbf{m,n}}\mathbf{B}_{\mathbf{n,k}} = \mathbf{C}_{\mathbf{m,k}}$$

# Multiplication of Two Matrices (Continuation)

An element in the product matrix is obtained by summing successive products of elements in the row of the first with elements of the column of the second.

$$c_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$$

# Division of Matrices ?

There is no such thing as division of matrices. However, *matrix inversion* can be viewed in some sense as a procedure similar to division. This process will be considered later.

Example: Determine **C = A + B** for the matrices shown below.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 9 & x \\ 6 & -3 & y \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 7 & 11 & 1+x \\ 2 & 2 & 6+y \end{bmatrix}$$

Example: Determine **D = A - B** for the matrices shown below.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ -4 & 5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 9 & x \\ 6 & -3 & y \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} -1 & -7 & 1-x \\ -10 & 8 & 6-y \end{bmatrix}$$

# Matrix Addition and Subtraction

Matrix addition and subtraction with MATLAB are achieved in the same manner as with scalars **provided** that the matrices have the same size. Typical expressions are shown below.

C = A + B

D = A - B

Example: Determine **C=AB**. A matrisinin sütün sayısı, B matrisinin satır sayısına eşit olmalıdır. A matrisinin satır sayısı, B matrisinin sütun sayısına eşit olmalıdır.

$$\mathbf{A} = \mathbf{A}_{2,3} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \qquad \mathbf{B} = \mathbf{B}_{3,2} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

# Example: Continuation.

$$c_{11} = (2)(2) + (-3)(7) + (5)(3) = 4 - 21 + 15 = -2$$

$$c_{12} = (2)(1) + (-3)(-4) + (5)(1) = 2 + 12 + 5 = 19$$

$$c_{21} = (-1)(2) + (4)(7) + (6)(3) = -2 + 28 + 18 = 44$$

$$c_{22} = (-1)(1) + (4)(-4) + (6)(1) = -1 - 16 + 6 = -11$$

$$\mathbf{C} = \begin{bmatrix} -2 & 19 \\ 44 & -11 \end{bmatrix}$$

# Example: Determine **D=BA**.

$$\mathbf{A} = \mathbf{A}_{2,3} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \qquad \mathbf{B} = \mathbf{B}_{3,2} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{BA} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 16 \\ 18 & -37 & 11 \\ 5 & -5 & 21 \end{bmatrix}$$

# Determinants

The determinant of a matrix **A** can be determined only for a *square* matrix. It is a *scalar* value. Various representations are shown as follows:

$$\det(\mathbf{A}) \qquad \left|\mathbf{A}\right| \qquad \triangle$$

# Determinant of 2 x 2 Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\det(\mathbf{A}) = a_{11}a_{22} - a_{12}a_{21}$$

# Determinants of Higher-Order

For determinants of matrices of higher order than 2 x 2, the process can become tedious. There are many “tricks”, but some are useful only when the matrix has simple numbers. The text provides a procedure based on minors and cofactors, but since the ultimate goal is to use MATLAB, that procedure will not be covered on these slides. Instead, formulas for the 3 x 3 case will be provided.

# Determinant of 3 x 3 Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{aligned} \det(\mathbf{A}) = a_{11}(a_{22}a_{33} - a_{23}a_{32}) \\ +a_{12}(-a_{21}a_{33} + a_{23}a_{31}) \\ +a_{13}(a_{21}a_{32} - a_{22}a_{31}) \end{aligned}$$

# Singular Matrix

If det(**A**)=0, the matrix is said to be *singular*. If the matrix represents the coefficients of a set of simultaneous equations, it means that the equations are not independent of each other and cannot be solved uniquely.

# Example: Determine the determinant of the matrix shown below.

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$\det(\mathbf{A}) = \begin{vmatrix} 3 & 2 \\ -4 & 5 \end{vmatrix} = (3)(5) - (2)(-4)$$

$$= 15 + 8 = 23$$

# Identity Matrix

$$\mathbf{I} = \begin{bmatrix} 1\;0\;0 \ldots. 0 \\ 0\;1\;0 \ldots. 0 \\ 0\;0\;1 \ldots. 0 \\ : \qquad\qquad\qquad : \\ 0\;0\;0\; \ldots. 1 \end{bmatrix}$$

# Inverse Matrix

The inverse of a matrix **A** is denoted by $\mathbf{A}^{-1}$ and is defined by the equation that follows.

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

# Inverse of a 2 x 2 Matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}}{\det(\mathbf{A})}$$

$$= \begin{bmatrix} \dfrac{a_{22}}{\det(\mathbf{A})} & \dfrac{-a_{12}}{\det(\mathbf{A})} \\ \dfrac{-a_{21}}{\det(\mathbf{A})} & \dfrac{a_{11}}{\det(\mathbf{A})} \end{bmatrix}$$

Example: Determine the inverse of the matrix **A** below.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\det(\mathbf{A}) = (2)(5) - (3)(4) = -2$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}}{-2} = \begin{bmatrix} -2.5 & 1.5 \\ 2 & -1 \end{bmatrix}$$

Example: The inverse of **A** below is developed in the text.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} -0.25 & -0.2 & 0.35 \\ 0.5 & 0.2 & -0.1 \\ -0.25 & 0.2 & 0.15 \end{bmatrix}$$

# Simultaneous Linear Equations

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \ldots . a_{1m}x_m = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots . a_{2m}x_m = b_2 \\
&\vdots \qquad\qquad\qquad\qquad\qquad \vdots \qquad \vdots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots . a_{mm}x_m = b_m
\end{aligned}
$$

# Matrix Form of Simultaneous Linear Equations

$$\begin{bmatrix} a_{11} & a_{12} \ldots\ldots a_{1m} \\ a_{21} & a_{22} \ldots\ldots a_{2m} \\ : & \quad : \\ a_{m1} & a_{m2} \ldots\ldots a_{mm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ : \\ b_m \end{bmatrix}$$

# Define variables as follows:

$$\mathbf{A} = \begin{bmatrix} a_{11} \; a_{12} \ldots. a_{1m} \\ a_{21} \; a_{22} \ldots. a_{2m} \\ \vdots \qquad\qquad \vdots \\ a_{m1} \; a_{m2} \ldots. a_{mm} \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

# Matrix Solution Development

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\mathbf{I}\mathbf{x} = \mathbf{x}$$

The general form and the final solution follow.

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

# Example: Use matrices to solve the simultaneous equations below.

$$x_1 + 2x_2 - x_3 = -8$$

$$-x_1 + x_2 + 3x_3 = 7$$

$$3x_1 + 2x_2 + x_3 = 4$$

# Example: Continuation.

$$\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ 4 \end{bmatrix}$$

$$\mathbf{Ax} = \mathbf{b}$$

# Example: Continuation.

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.25 & -0.2 & 0.35 \\ 0.5 & 0.2 & -0.1 \\ -0.25 & 0.2 & 0.15 \end{bmatrix} \begin{bmatrix} -8 \\ 7 \\ 4 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

# Transformation of Linear Variables

$$y_1 = b_{11}x_1 + b_{12}x_2$$

$$y_2 = b_{21}x_1 + b_{22}x_2$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\mathbf{y} = \mathbf{B}\mathbf{x}$$

# Transformation Continuation

$$z_1 = a_{11}y_1 + a_{12}y_2$$

$$z_2 = a_{21}y_1 + a_{22}y_2$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\mathbf{z} = \mathbf{A}\mathbf{y}$$

# Transformation Continuation

$$\mathbf{z} = \mathbf{A}\mathbf{y}$$

$$\mathbf{y} = \mathbf{B}\mathbf{x}$$

$$\mathbf{z} = \mathbf{A}\mathbf{B}\mathbf{x} = \mathbf{C}\mathbf{x}$$

$$\mathbf{C} = \mathbf{A}\mathbf{B}$$

Example: For the system of equations provided below, determine the *z* values in terms of the *x* values.

$$y_1 = 2x_1 - 3x_2$$

$$y_2 = 4x_1 - 2x_2$$

$$z_1 = 5y_1 - 2y_2$$

$$z_2 = 4y_1 + 3y_2$$

# Example: Continuation.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

# Example: Continuation.

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 & -11 \\ 20 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

# Error Messages

MATLAB has many error messages that indicate problems with operations. If the matrices have different sizes, the message is

```
??? Error using ==> ±
Matrix dimensions must agree.
```

# Matrix Multiplication

Matrix multiplication with MATLAB is achieved in the same manner as with scalars **provided** that the number of columns of the first matrix is equal to the number of rows of the second matrix. A typical expression is

```
>> E = A*B
```

# Örnek

```
A=[3 2 1; -4 5 6]
B=[4 9 1; 6 -3 1; 3 4 0]
C=A*B
```

```
A =

     3     2     1
    -4     5     6


B =

     4     9     1
     6    -3     1
     3     4     0


C =

    27    25     5
    32   -27     1
```

# Array Multiplication

There is another form of multiplication of matrices in which it is desired to multiply corresponding elements in a fashion similar to that of addition and subtraction. This operation arises frequently with MATLAB, and we will hereafter refer to the process as the **array product** to distinguish it from the standard matrix multiplication form.

# Array Multiplication Continuation

For the array product to be possible, the two matrices must have the same size, as was the case for addition and subtraction. The resulting array product will have the same size. If **F** represents the resulting matrix, a given element of **F**, denoted by $f_{ij}$ is determined by the corresponding product from the two matrices as

$$f_{ij} = a_{ij} b_{ij}$$

# Örnek

```
A=[3 2 1]
B=[4; 6; 3]
C=A*B
```

```
A =
     3     2     1
B =
     4
     6
     3
C =
    27
```

# MATLAB Array Multiplication

To form an array product in MATLAB, a period must be placed after the first variable. The operation is commutative. The following two operations produce the same result. Skaler çarpım söz konusudur.Marislerin karşılıklı elemanları çarpılır.

```
F=A.*B
F=B.*A
```

# Örnek

```
A=[3 2 1]
B=[4 6 3]
C=A.*B
```

```
A =
     3     2     1

B =
     4     6     3

C =
    12    12     3
```

# MATLAB Array Multiplication Continuation

If there are more than two matrices for which array multiplication is desired, the periods should follow all but the last one in the expression; e. g., A.*B.*C in the case of three matrices. Alternately, nesting can be used; e.g. (A.*B).*C for the case of three matrices.

# MATLAB Array Multiplication Continuation

The array multiplication concept arises in any operation in which the command could be “confused” for a standard matrix operation. For example, suppose it is desired to form a matrix B from a matrix A by raising each element of A to the 3rd power, The MATLAB command is

```
>> B = A.^3
```

# Örnek

```
A=[3 2 1]
B=A.^3
```

```
A =

     3     2     1


B =

    27     8     1
```

# Determinant of a Matrix

The determinant of a square matrix in MATLAB is determined by the simple command **det(A)**. Thus, if **a** is to represent the determinant, we would type and enter

```
>> a = det(A)
```

Note that **a** is a scalar (1 x 1 "matrix").

# Örnek

```
A=[3]
B=det(A)
```

```
B =    3
```

```
A=[3 2; 1 -2]
B=det(A)
```

```
A =
     3     2
     1    -2

B =
    -8
```

# Inverse Matrix

The inverse of a square matrix in MATLAB is determined by the simple command **inv(A)**. Thus, if **B** is to represent the inverse of **A** , the command would be

```
B = inv(A)
```

# Örnek

```
A=[3 2; 1 -2]
B=inv(A)
C=A*B
D=B*A
```

```
A =
    3     2
    1    -2
B =
    0.2500    0.2500
    0.1250   -0.3750
```

```
C =
     1     0
     0     1

D =
     1     0
     0     1
```

# Simultaneous Equation Solution

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

MATLAB Format:

```
>> x = inv(A)*b
```

Alternate MATLAB Format:

```
>> x = A\b
```

Example 3-1. Enter the matrices below in MATLAB. They will be used in the next several examples.

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 5 \\ -1 & 4 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ 7 & -4 \\ 3 & 1 \end{bmatrix}$$

```
>> A = [2 -3 5; -1 4 6];
```

```
>> B = [2 1; 7 -4; 3 1];
```

## Example 3-2. Determine the transpose of B and denote it as C.

```
>> C = B'


C =

     2     7     3

     1    -4     1
```

The 3 x 2 matrix has been converted to a 2 x 3 matrix.

# Example 3-3. Determine the sum of A and C and denote it as D.

```
>> D = A + C


D =
     4     4     8
     0     0     7
```

# Example 3-4. Determine the product of A and B with A first.

```
>> A*B

ans =

    -2    19
    44   -11
```

# Example 3-5. Determine the product of B and A with B first.

```
>> B*A

ans =

     3    -2    16
    18   -37     11
     5    -5    21
```

# Example 3-6. Determine the array product of A and C and denote it as E.

```
>> E = A.*C

E =

     4   -21    15
    -1   -16     6
```

# Example 3-7. Enter the matrix **A**. It will be used in several examples.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

```
>> A = [1 2 -1; -1 1 3; 3 2 1]
A =
     1     2    -1
    -1     1     3
     3     2     1
```

# Example 3-7. Continuation. Determine the determinant of A and denote it as a.

```
>> a = det(A)

a =

    20
```

# Example 3-8. Determine the inverse matrix of A and denote it as Ainv.

```
>> Ainv = inv(A)

Ainv =

   -0.2500   -0.2000    0.3500
    0.5000    0.2000   -0.1000
   -0.2500    0.2000    0.1500
```

Example: Use MATLAB to solve the simultaneous equations below.

$$x_1 + 2x_2 - x_3 = -8$$

$$-x_1 + x_2 + 3x_3 = 7$$

$$3x_1 + 2x_2 + x_3 = 4$$

# Örnek

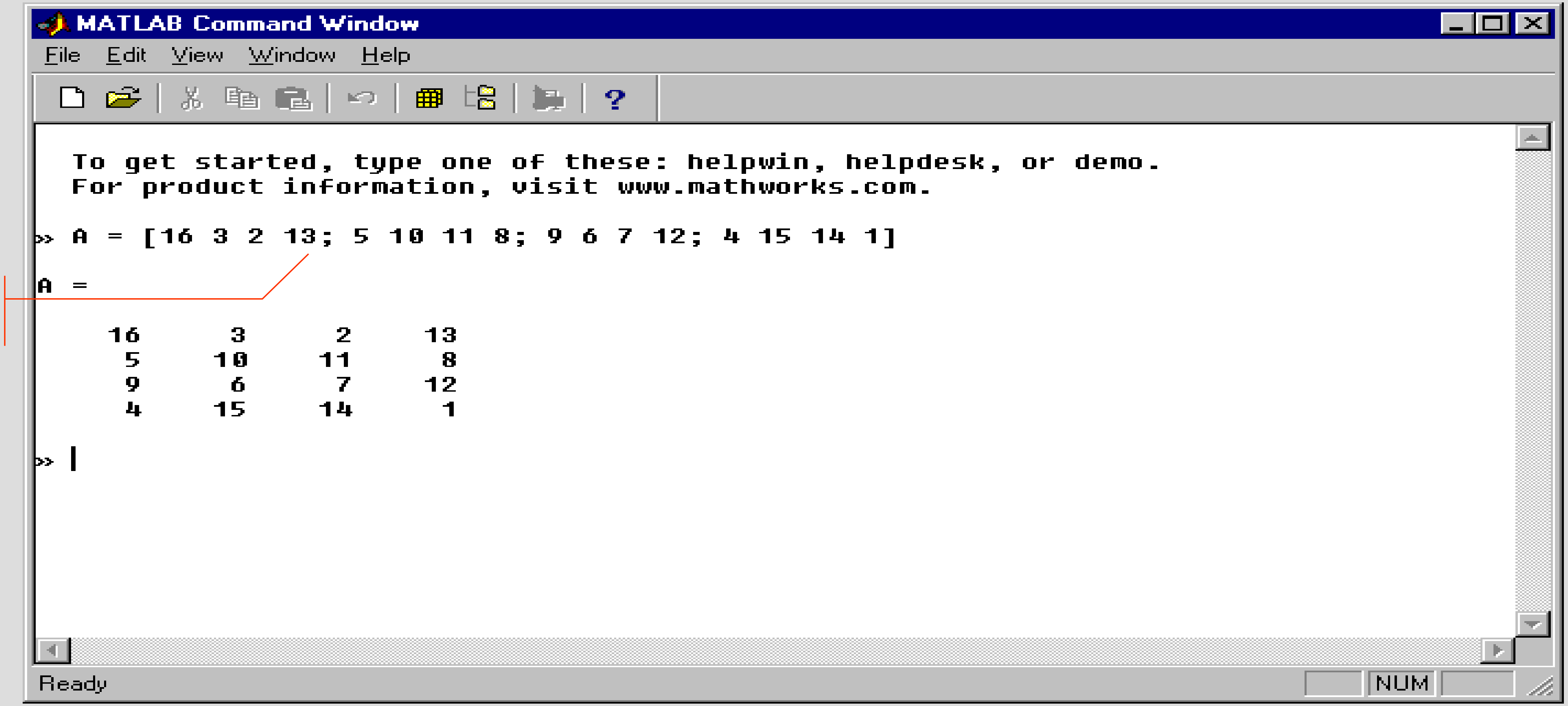
```
A=[1 2 -1; -1 1 3; 3 2 1]
B=[-8; 7;  4]
C=inv(A)*B
```

```
A =
     1     2    -1
    -1     1     3
     3     2     1


B =
    -8
     7
     4
```

```
C =
    2.0000
   -3.0000
    4.0000
```

# Matrices

- Entering matrices
  - explicitly

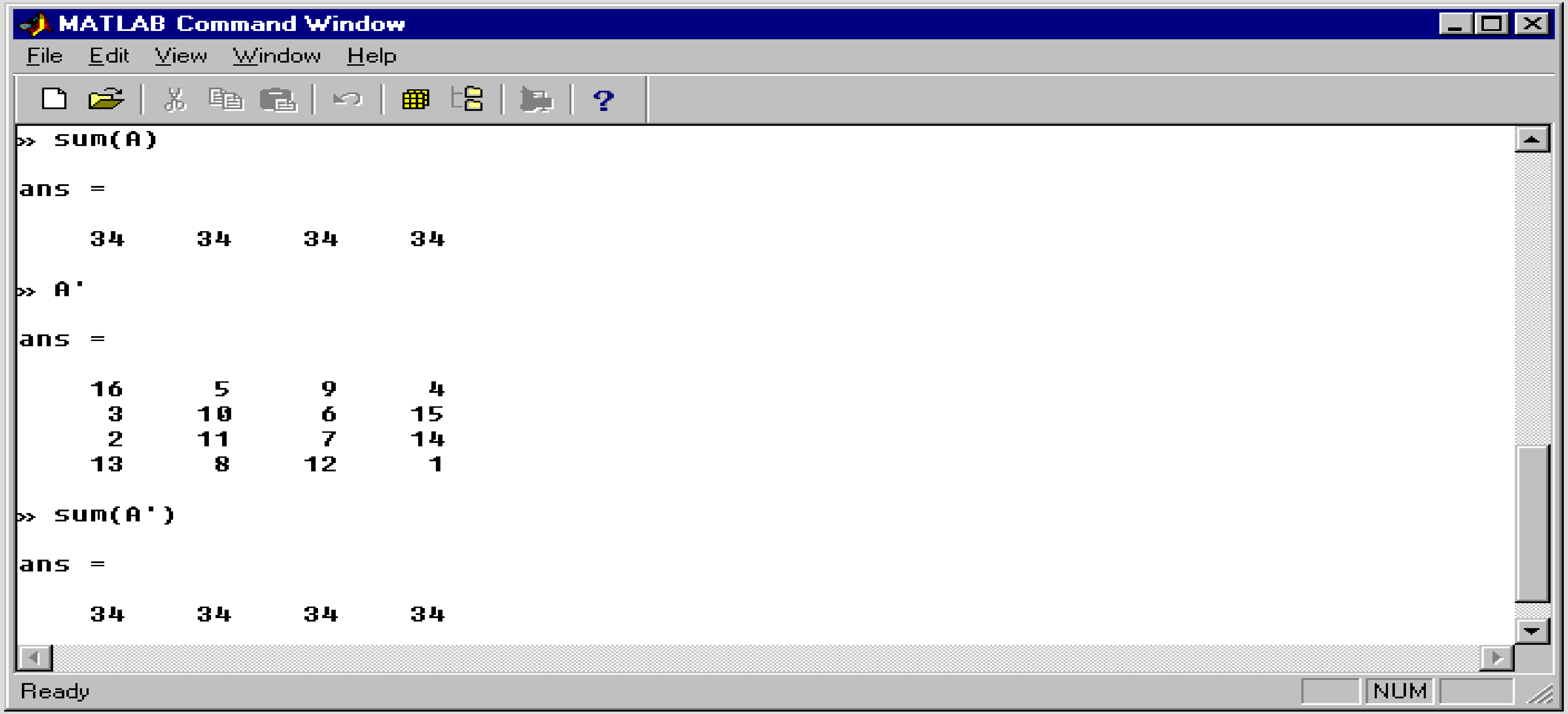


```
  To get started, type one of these: helpwin, helpdesk, or demo.
  For product information, visit www.mathworks.com.

>> A = [16 3 2 13; 5 10 11 8; 9 6 7 12; 4 15 14 1]

A =

    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    15    14     1

>> 
```

**semicolon**

# Matrices

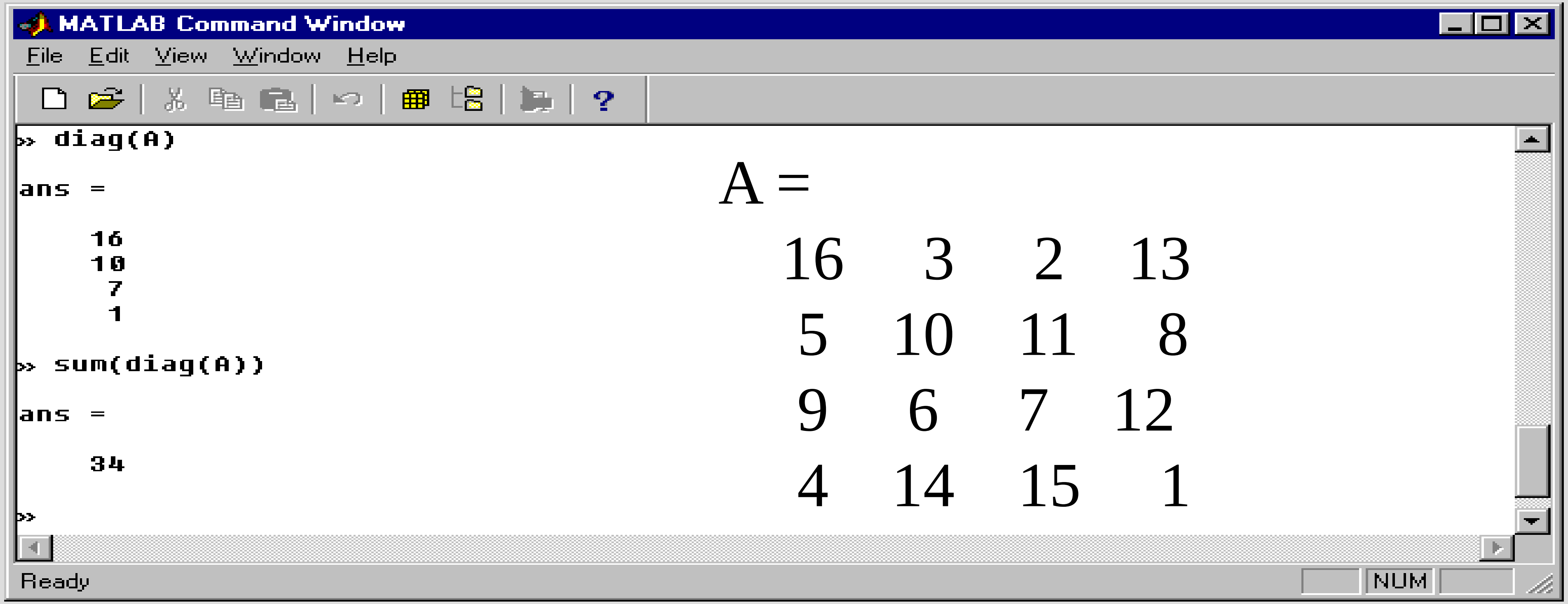
Sum komutu sütun değerleri topar, transpose’de satırlar ile sütüunlar ile yer değiştirir.



```
>> sum(A)

ans =

    34    34    34    34

>> A'

ans =

    16     5     9     4
     3    10     6    15
     2    11     7    14
    13     8    12     1

>> sum(A')

ans =

    34    34    34    34
```

# Matrices

Diagonal (diag), matrisin köşegenini oluşturan değerlerdir.

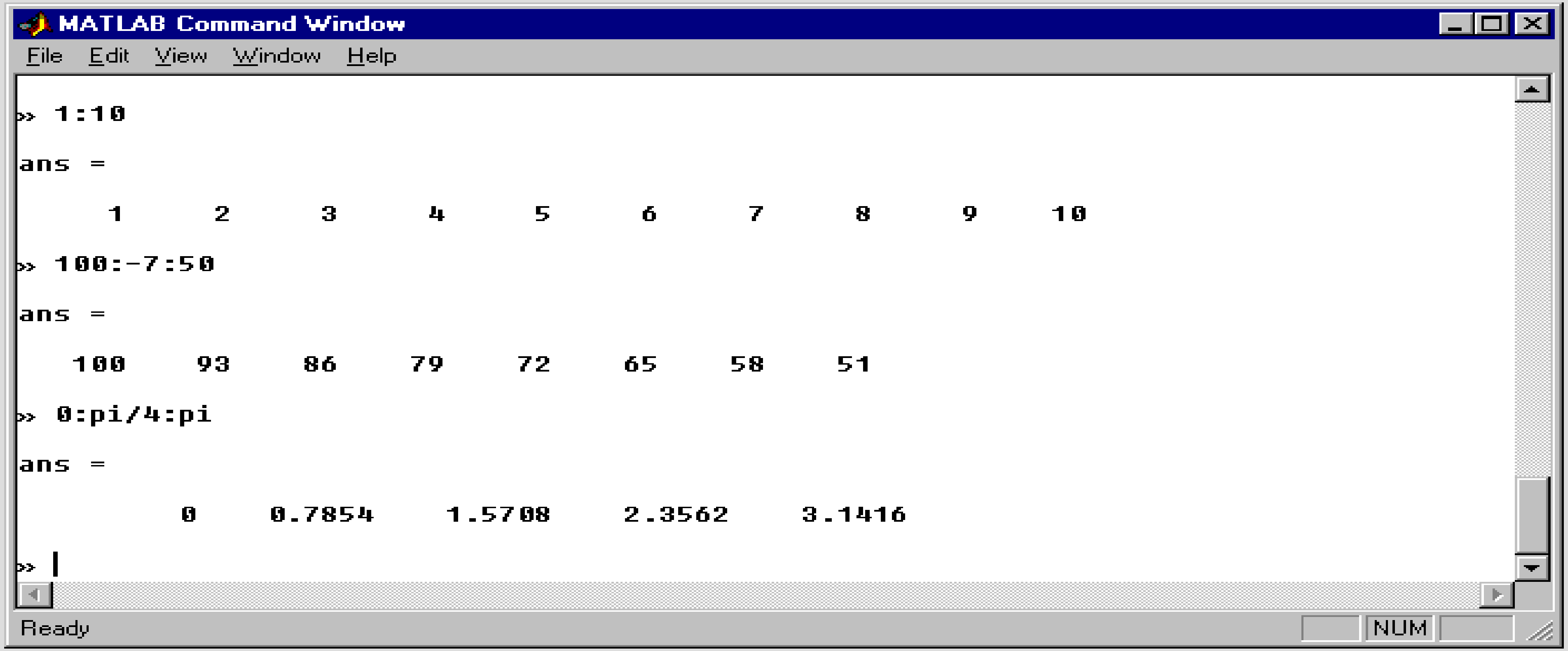


```
>> diag(A)

ans =

    16
    10
     7
     1

>> sum(diag(A))

ans =

    34

>>
```

$$A = \begin{matrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 14 & 15 & 1 \end{matrix}$$

# Matrices

Colon operator ' : '



```
>> 1:10

ans =

     1     2     3     4     5     6     7     8     9    10

>> 100:-7:50

ans =

   100    93    86    79    72    65    58    51

>> 0:pi/4:pi

ans =

         0    0.7854    1.5708    2.3562    3.1416

>> 
```

# Matrices: Colon operator ' : '

```
A=[16 3 2 13; 5 10 11 8;9 6 7 12;4 14 15 1]
c=size(A)
U=A(2,:)
```

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    14    15     1

c =    4     4

U =   5    10    11     8
```

# Matrices: Colon operator ' : '

```
A=[16 3 2 13; 5 10 11 8;9 6 7 12;4 14 15 1]

V=A(1,2:4)
```

```
A =

    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    14    15     1


V =

     3     2    13
```

# Matrices: Colon operator ' : `
# 2 ve 3. sütunlar

```
A=[16 3 2 13; 5 10 11 8;9 6 7 12;4 14 15 1]

d=A(:,2:3)
```

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    14    15     1

d =
     3     2
    10    11
     6     7
    14    15
```

# Matrices: Colon operator ' : `
## d=a(:,1:2) (A matrisinin tüm satırlarını ve 1. ve 2. sütunlarını alır)

```
A=[16 3 2 13; 5 10 11 8;9 6 7 12;4 14 15 1]

d=A(:,1:2)
```

```
d =


    16     3
     5    10
     9     6
     4    14
```

# Matrix multiplication ( * )

```
A=[16 3 2 13; 5 10 11 8;9 6 7 12;4 14 15 1]
Y=[3 5 4 1]'
B=A*Y
```

```
B =
    84
   117
    97
   143
```

```
A =
    16     3     2    13
     5    10    11     8
     9     6     7    12
     4    14    15     1

Y =
     3
     5
     4
     1
```

# Matrix multiplication ( * )

İki matrisin çarpılabilmesi için satır eleman sayısı ile sütun eleman sayısı eşit olmak zorunda, bu nedenaşağıdaki örnekte transpoese alındı.

```
A=[2 5 8]
B=[4 3 5]'
C=A*B
```

C=2*4+5*3+8*5=8+15+40=63

```
A =    2    5    8

B =
     4
     3
     5

C =   63
```

# Matrix multiplication ( .* )

İki matrisin çarpılabilmesi için satır eleman sayısı ile sütun eleman sayısı eşit olmak zorunda, bu nedenaşağıdaki örnekte transpoese alındı.

```
A=[2 5 8]
B=[4 3 5]
C=A.*B
```

```
A =     2     5     8

B =     4     3     5

C =     8    15    40
```

C=2*4  5*3 8*5

# Variables

## Scalar, vector, matrix

```
a=5
A=[400 300 20]
B=A'
C=[2 3 4 5; 1 3 6 7]
```

```
a =    5
A =   400   300    20

B =
   400
   300
    20

C =
     2     3     4     5
     1     3     6     7
```

a:skaler

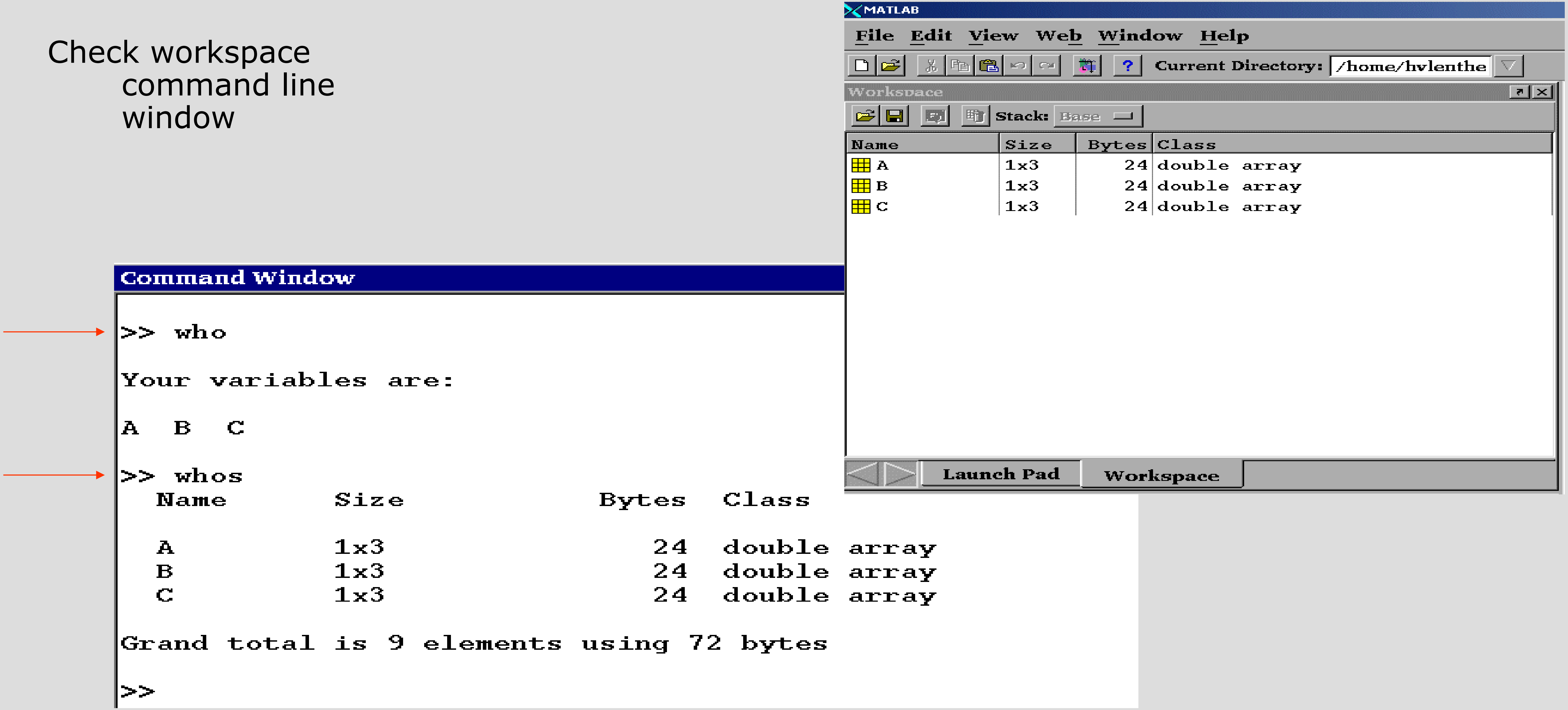
A: Satır vektörü

B: Sütun vektörü

C: Matris

# Variables

Check workspace
command line
window



| Name | Size | Bytes | Class |
|---|---|---|---|
| A | 1x3 | 24 | double array |
| B | 1x3 | 24 | double array |
| C | 1x3 | 24 | double array |

```
>> who

Your variables are:

A   B   C

>> whos
  Name        Size              Bytes  Class

  A           1x3                  24  double array
  B           1x3                  24  double array
  C           1x3                  24  double array

Grand total is 9 elements using 72 bytes

>>
```

# Generating matrices

```
Z=zeros(4,4)
```

```
Z =


     0     0     0     0
     0     0     0     0
     0     0     0     0
     0     0     0     0
```

# Generating matrices

```
B=ones(4,4)
```

```
B =

     1     1     1     1
     1     1     1     1
     1     1     1     1
     1     1     1     1
```

# Generating matrices

0 ile 1 arasında rasgele sayı üretme `d=rand()`

```
d =    0.8147
```

0 ile 1 arasında 10 adet rasgele sayı üretme (dizi) `N=fix(10*rand(1,10))`

0 ile 1 arasında (5,10) adet rasgele sayı üretme (Matris)

`N=fix(10*rand(5,10))`

```
N =

     5     3     1     9     9     2     9     8     7     7
     5     5     1     9     3     4     5     0     6     1
     1     4     2     4     1     0     0     0     4     6
     8     0     4     4     7     1     2     1     5     1
     6     2     0     3     3     9     3     6     2     3
```

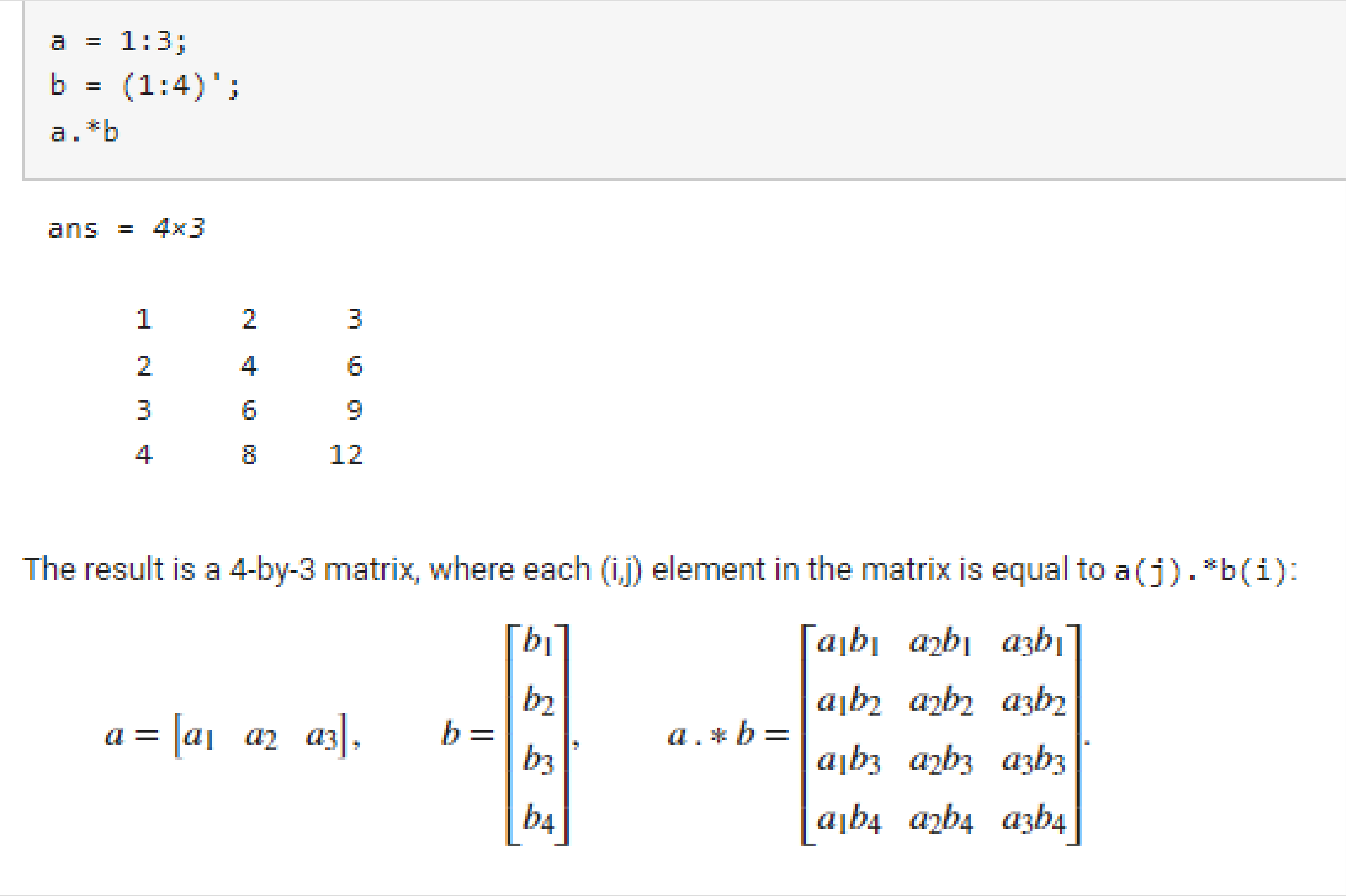
# Multiplication: times, .*

```
C = A.*B
C = times(A,B)
```

Örnek:

```
A = [1 0 3];
B = [2 3 7];
C = A.*B
2     0    21
```

```
A = [1 0 3; 5 3 8; 2 4 6];
B = [2 3 7; 9 1 5; 8 8 3];
C = A.*B
C = 3×3

     2     0    21
    45     3    40
    16    32    18
```



```
a = 1:3;
b = (1:4)';
a.*b
```

```
ans = 4×3

     1     2     3
     2     4     6
     3     6     9
     4     8    12
```

The result is a 4-by-3 matrix, where each (i,j) element in the matrix is equal to `a(j).*b(i)`:

$$a = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}, \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}, \qquad a\,.\!*\,b = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}.$$

```
A = [1 1 0 0];
B = [1; 2; 3; 4];
```

Multiply A times B.

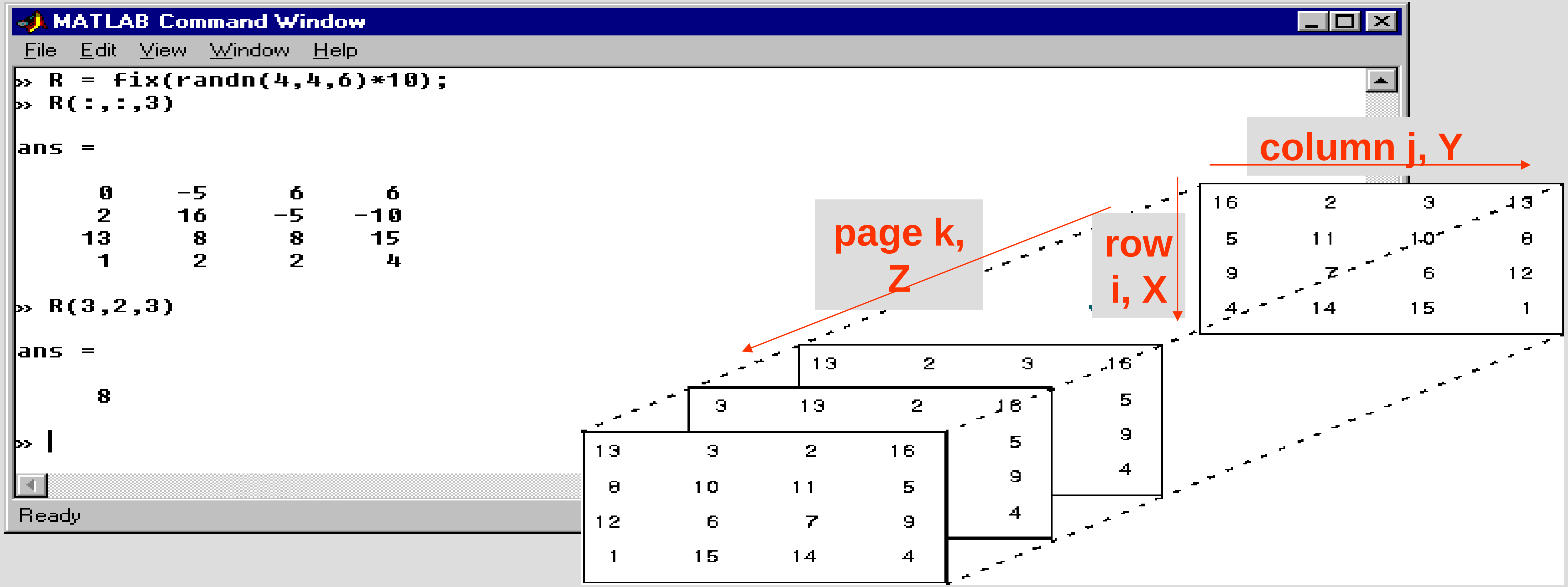
```
C = A*B
C = 3
```

The result is a 1-by-1 scalar, also called the dot product or inner product of the vectors A and B. Alternatively, you can calculate the dot product A·B with the syntax dot(A,B).

Multiply B times A.

```
C = B*A
C = 4×4
    1    1    0    0
    2    2    0    0
    3    3    0    0
    4    4    0    0
```

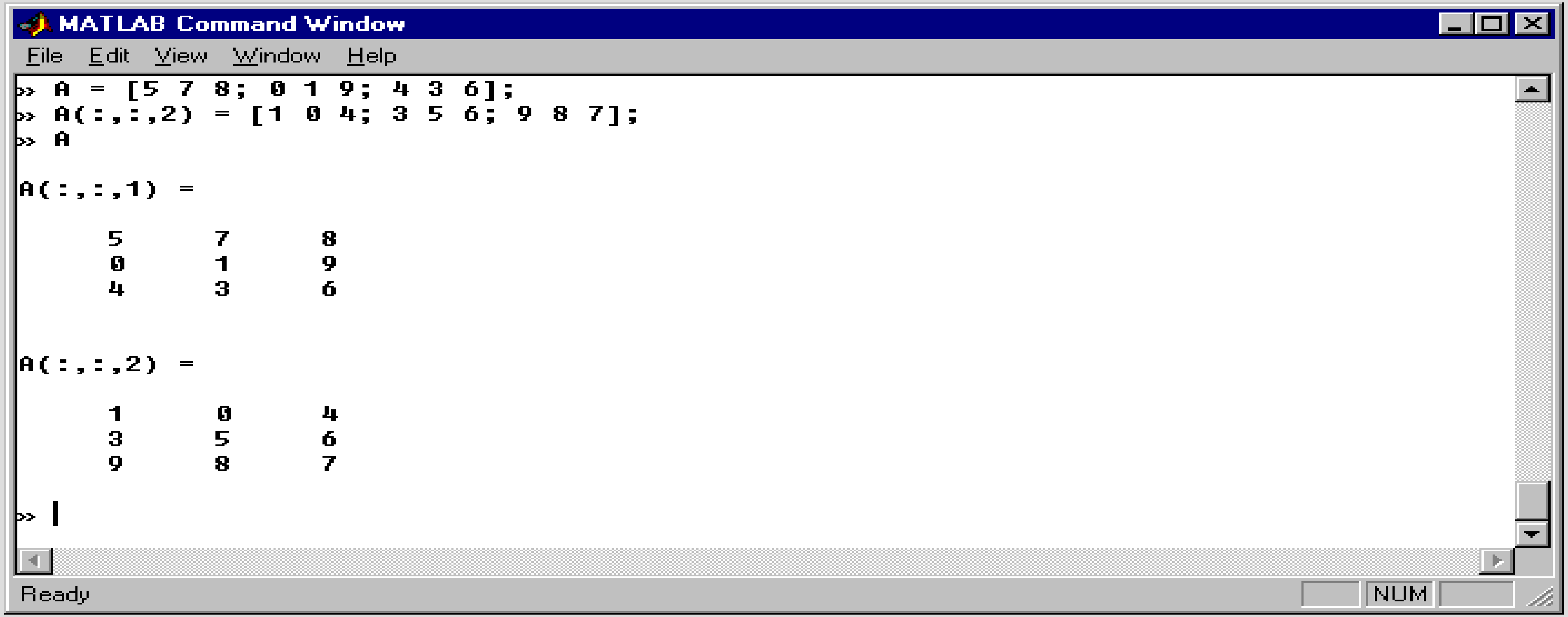
# Multidimensional matrices



```
>> R = fix(randn(4,4,6)*10);
>> R(:,:,3)

ans =

     0    -5     6     6
     2    16    -5   -10
    13     8     8    15
     1     2     2     4

>> R(3,2,3)

ans =

     8

>> |
```

# Multidimensional matrices

## Generate a 3D array manually



```
>> A = [5 7 8; 0 1 9; 4 3 6];
>> A(:,:,2) = [1 0 4; 3 5 6; 9 8 7];
>> A

A(:,:,1) =

     5     7     8
     0     1     9
     4     3     6


A(:,:,2) =

     1     0     4
     3     5     6
     9     8     7

>> 
```

# Advanced indexing

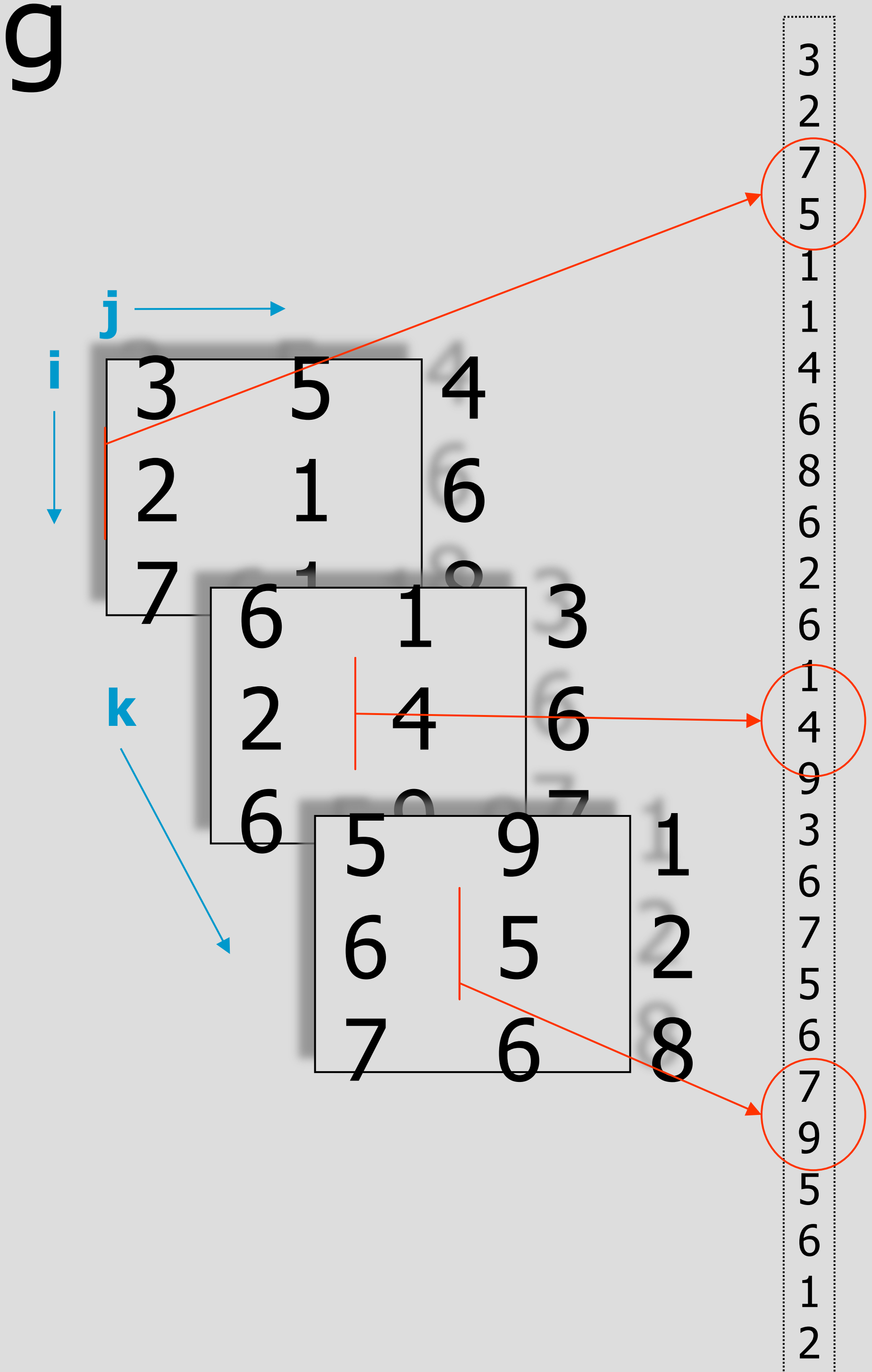
Matrices are always stored as columns

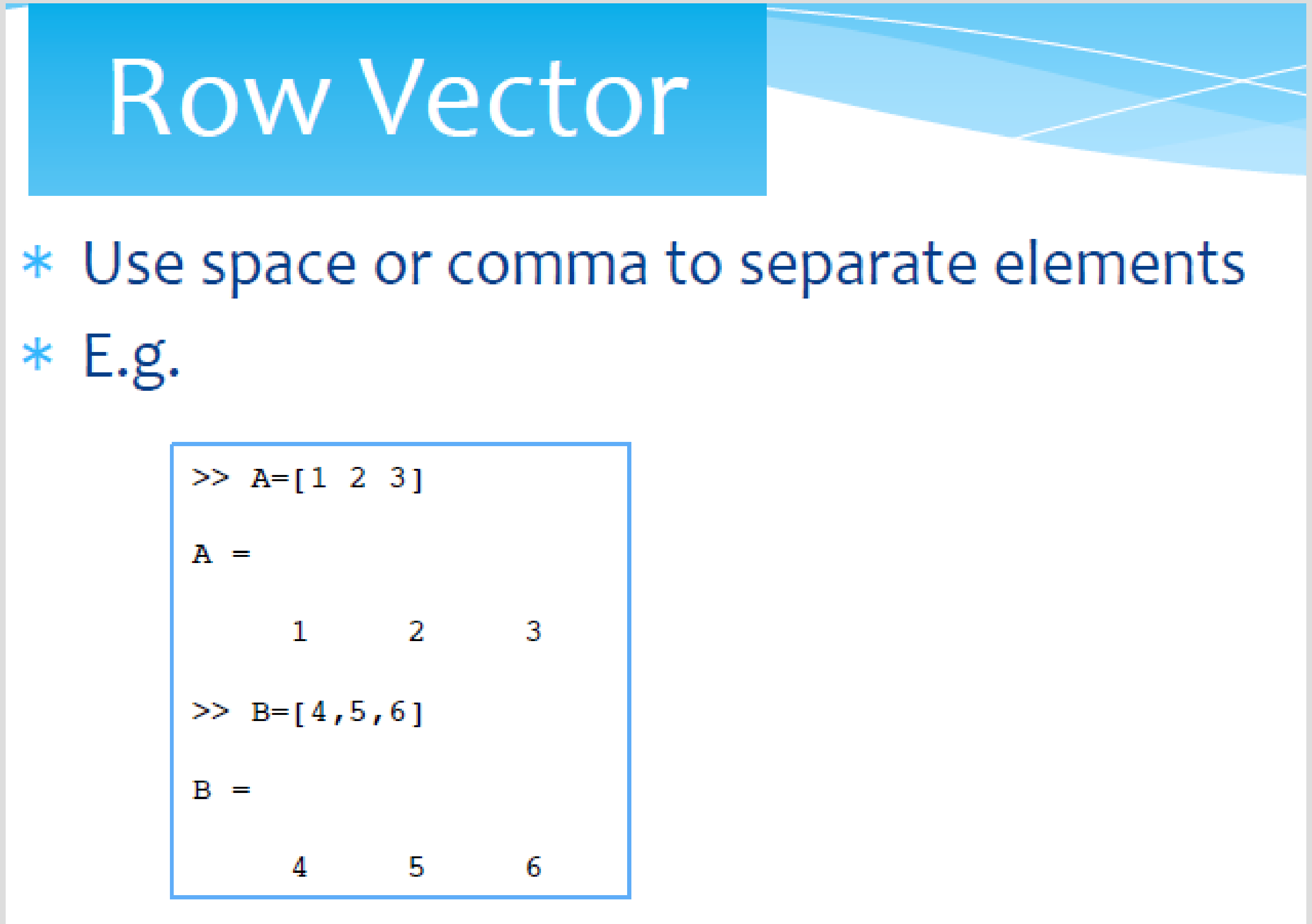
subscripts (i,j,k)

array dimension [$d_1$ $d_2$ $d_3$]

if A is of dimension 3 x 3 x 3

then A(3,3,3) = A(27) = 8

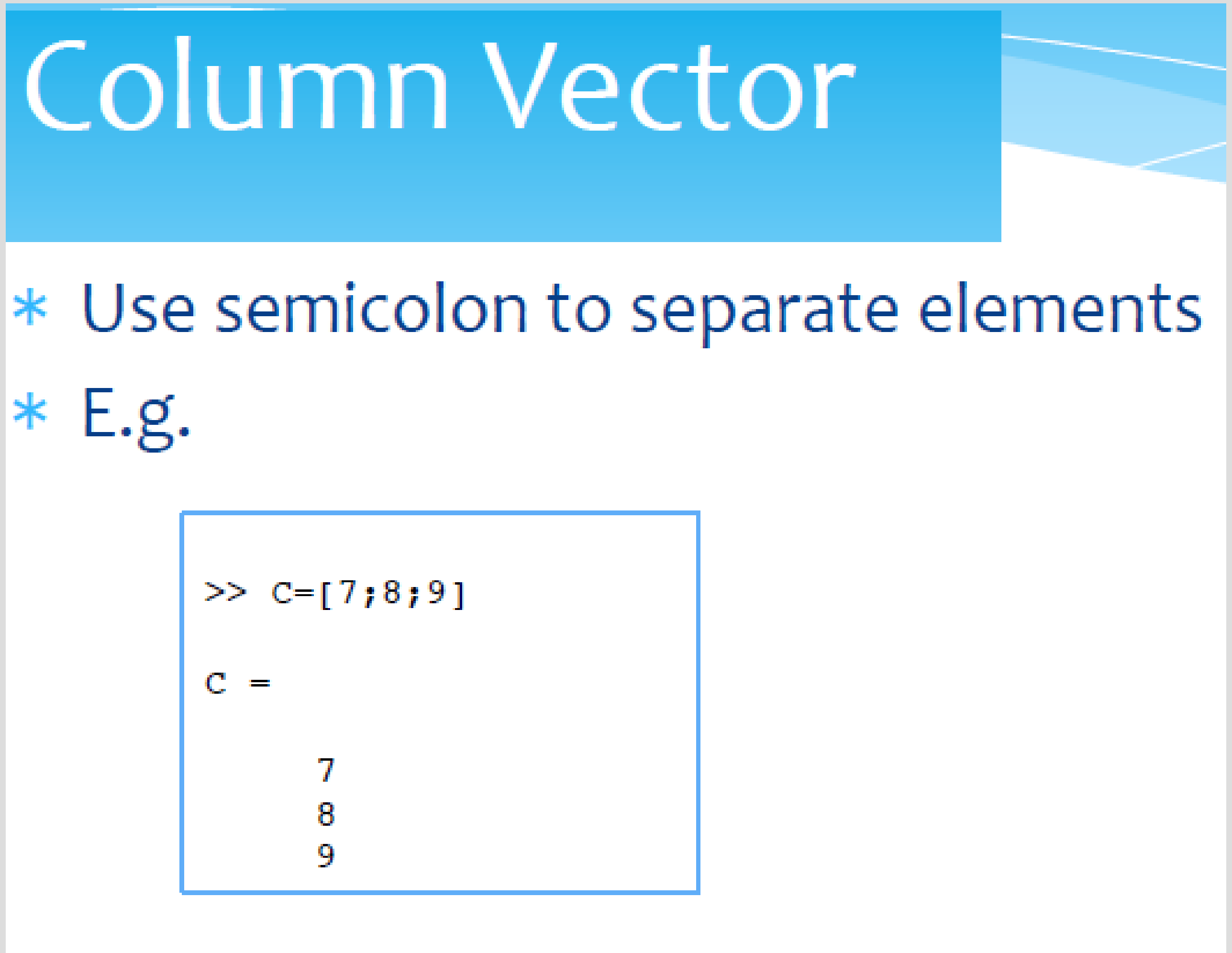




# Row Vector

- Use space or comma to separate elements
- E.g.

```
>> A=[1 2 3]

A =

     1     2     3

>> B=[4,5,6]

B =

     4     5     6
```



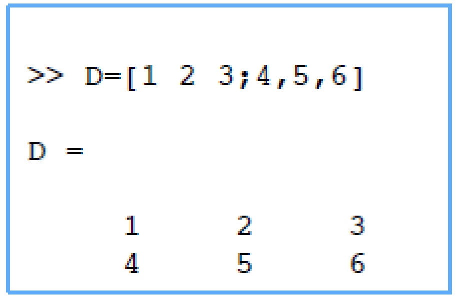
# Column Vector

- Use semicolon to separate elements
- E.g.

```
>> C=[7;8;9]

C =

     7
     8
     9
```

# Matrix

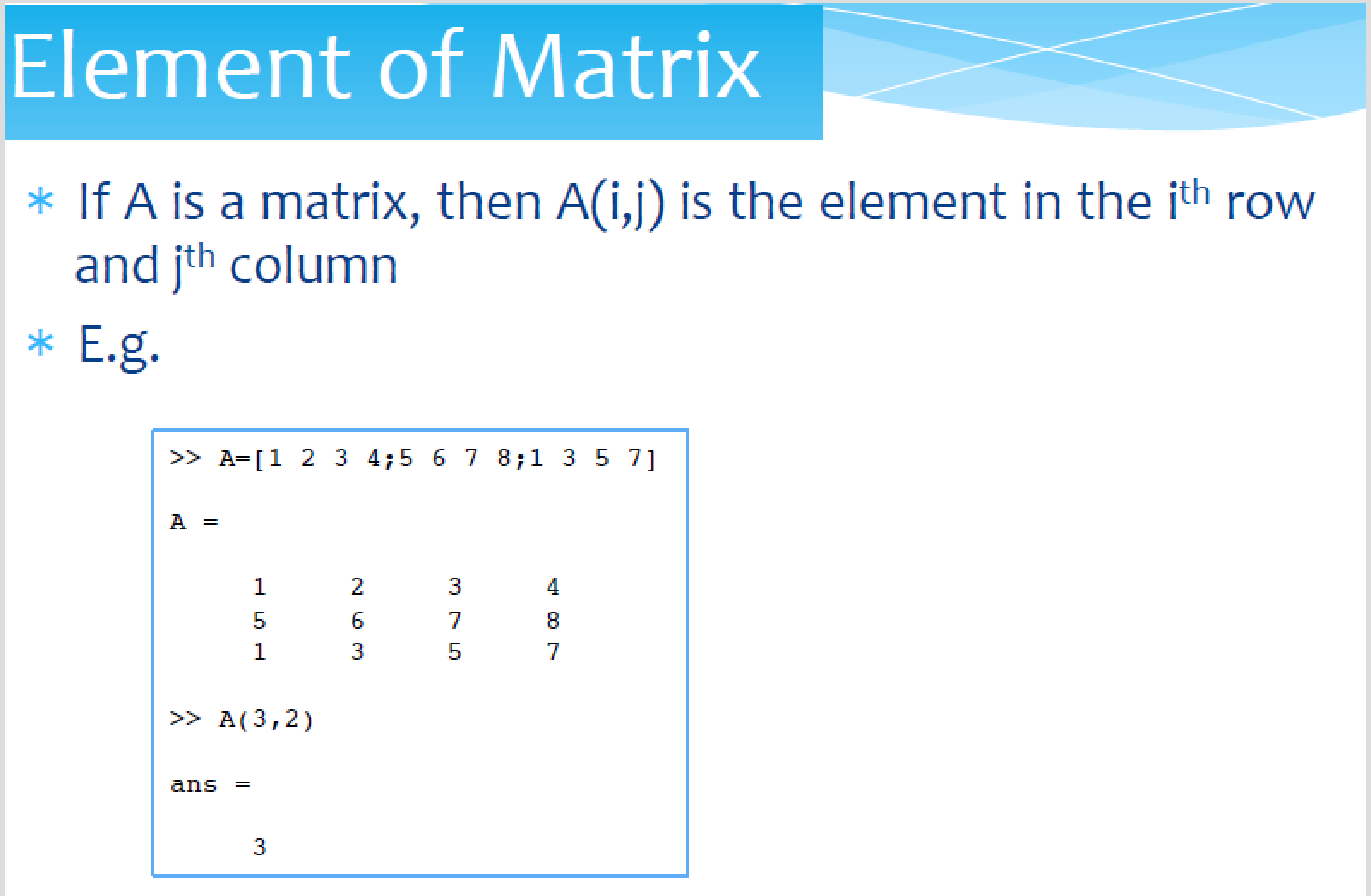
- Matrix can be viewed as a column vector of row vectors
- Use semicolon to separate rows, and use space or comma to separate elements in a row
- E.g.



```
>> D=[1 2 3;4,5,6]

D =

     1     2     3
     4     5     6
```

# Vector and Matrix

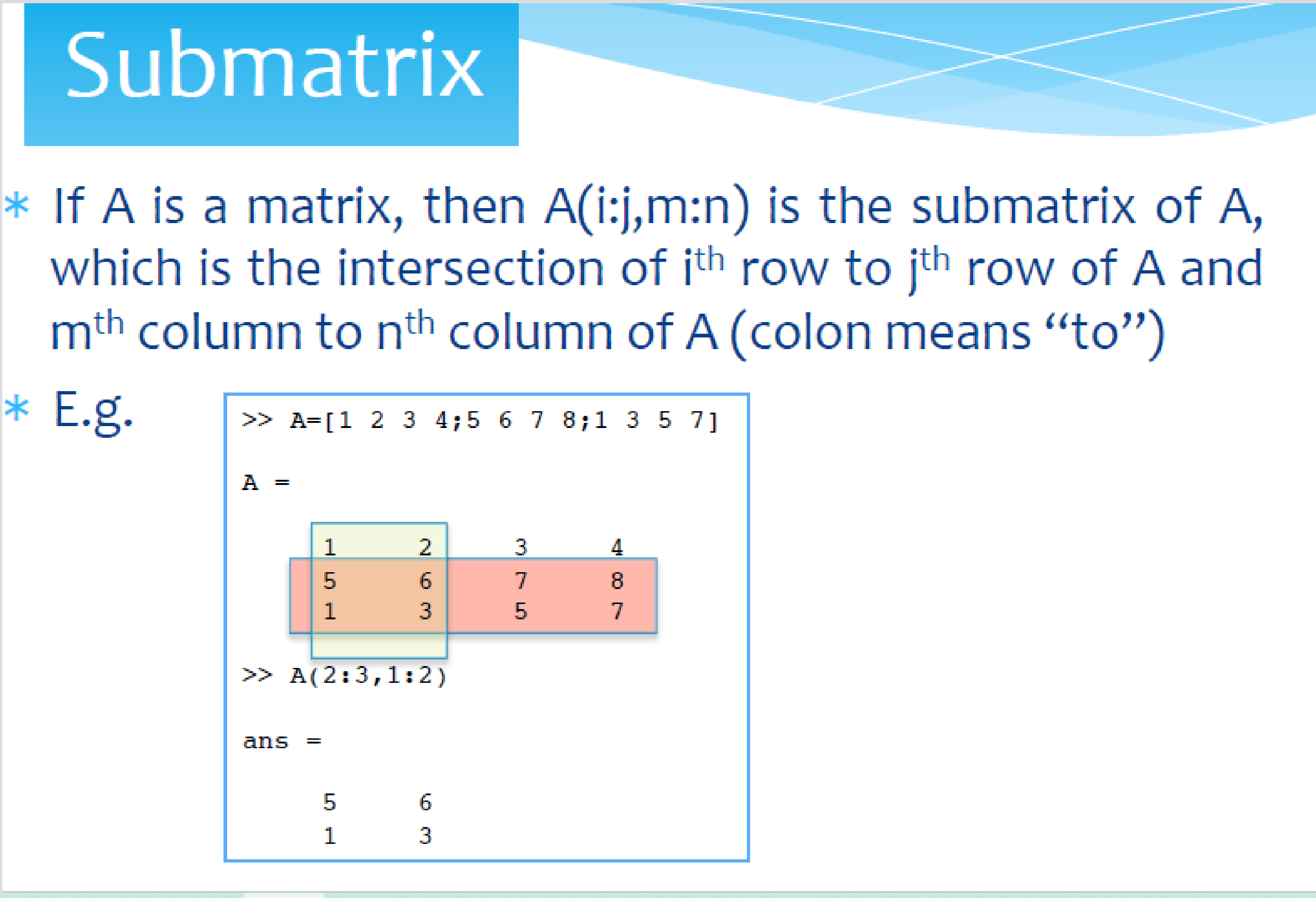
- Row vector is a special case of matrix: only one row
- Column vector is a special case of matrix: only one column
- Scalar is also a special case of matrix: only one row and only one column



# Element of Matrix

- If A is a matrix, then A(i,j) is the element in the i$^{th}$ row and j$^{th}$ column
- E.g.

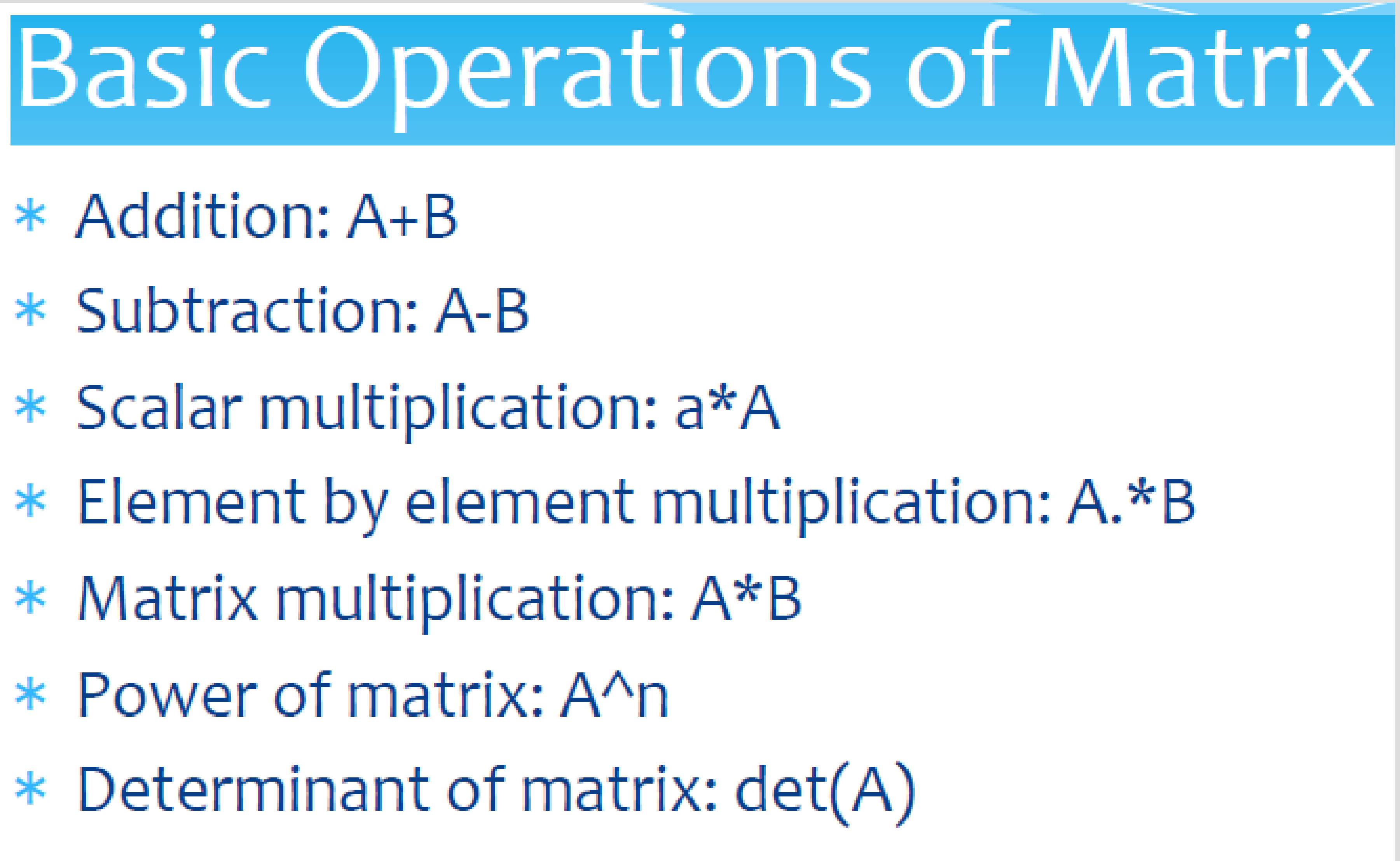
```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]

A =

     1     2     3     4
     5     6     7     8
     1     3     5     7

>> A(3,2)

ans =

     3
```



# Submatrix

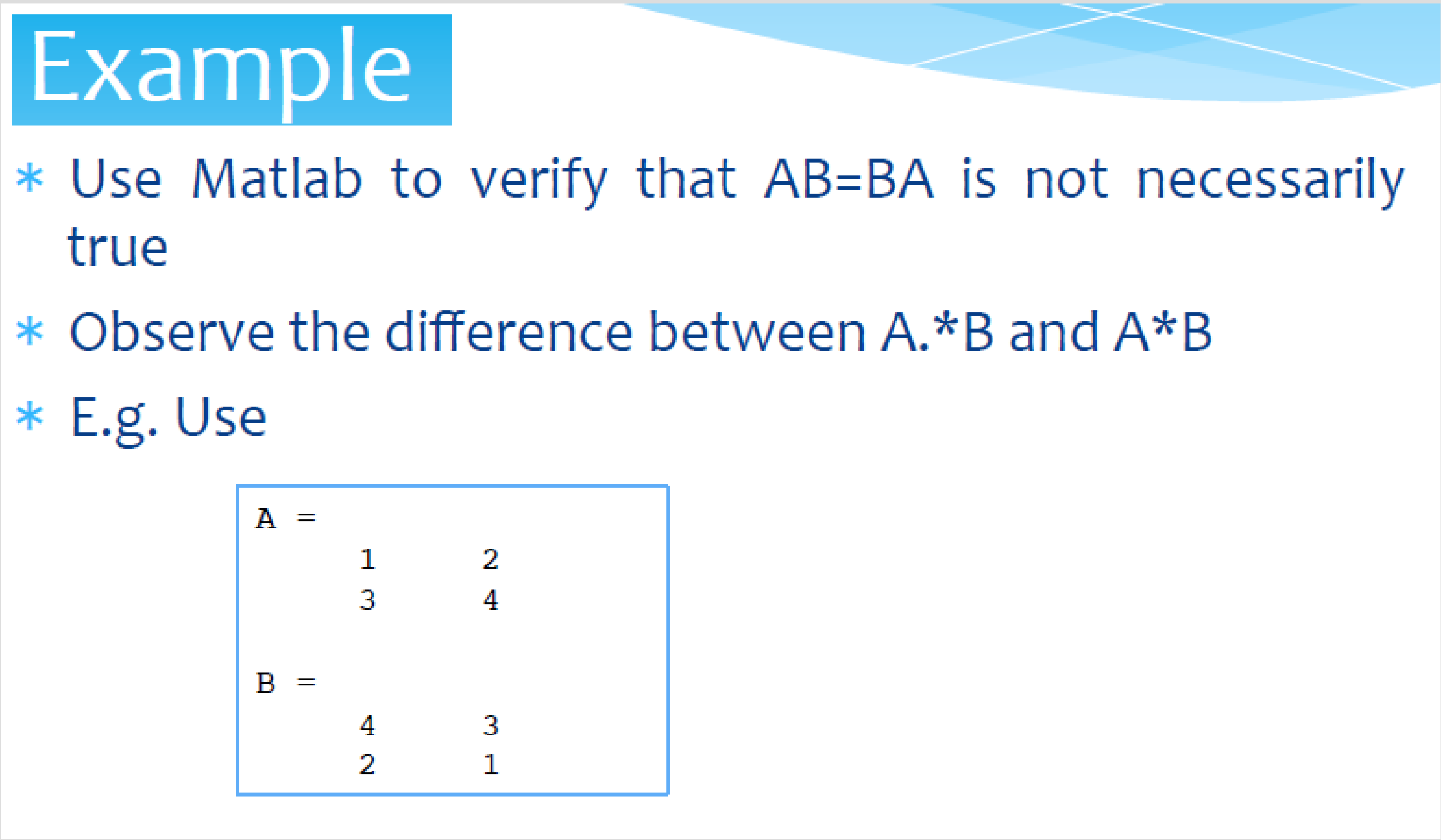
- If A is a matrix, then A(i:j,m:n) is the submatrix of A, which is the intersection of i$^{th}$ row to j$^{th}$ row of A and m$^{th}$ column to n$^{th}$ column of A (colon means “to”)
- E.g.

```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]

A =

     1     2     3     4
     5     6     7     8
     1     3     5     7

>> A(2:3,1:2)

ans =

     5     6
     1     3
```



# Basic Operations of Matrix

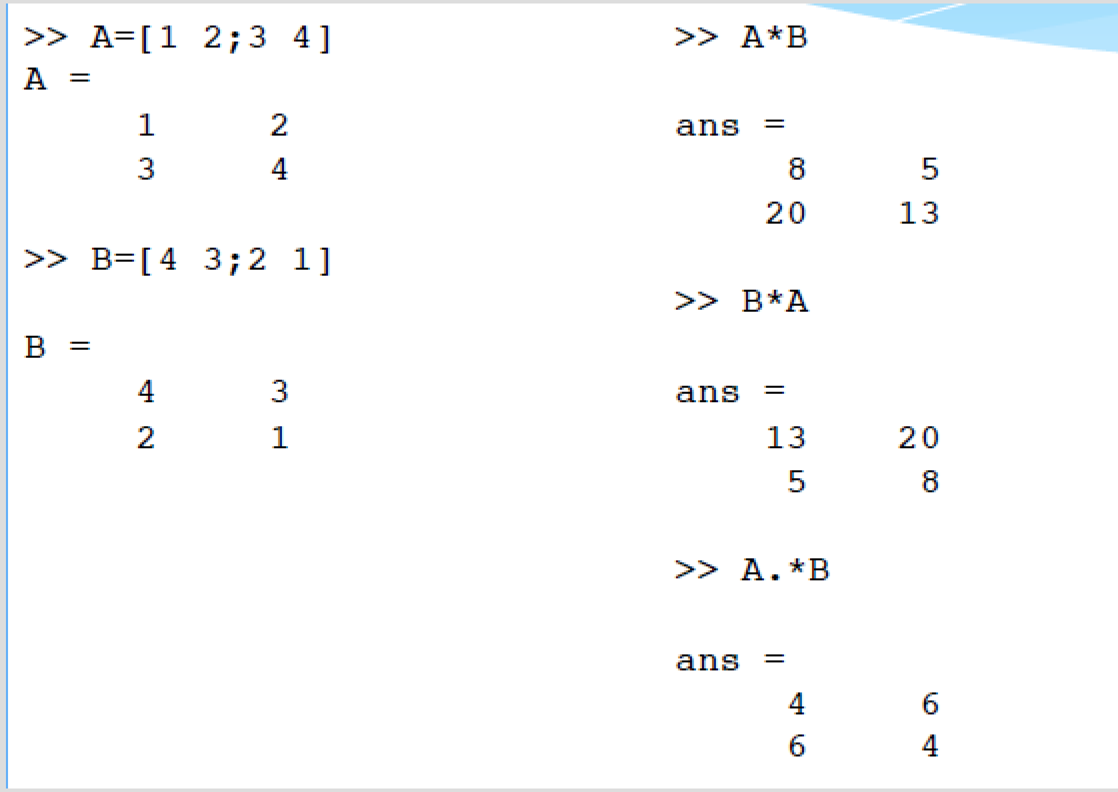
- Addition: A+B
- Subtraction: A-B
- Scalar multiplication: a*A
- Element by element multiplication: A.*B
- Matrix multiplication: A*B
- Power of matrix: A^n
- Determinant of matrix: det(A)



# Example

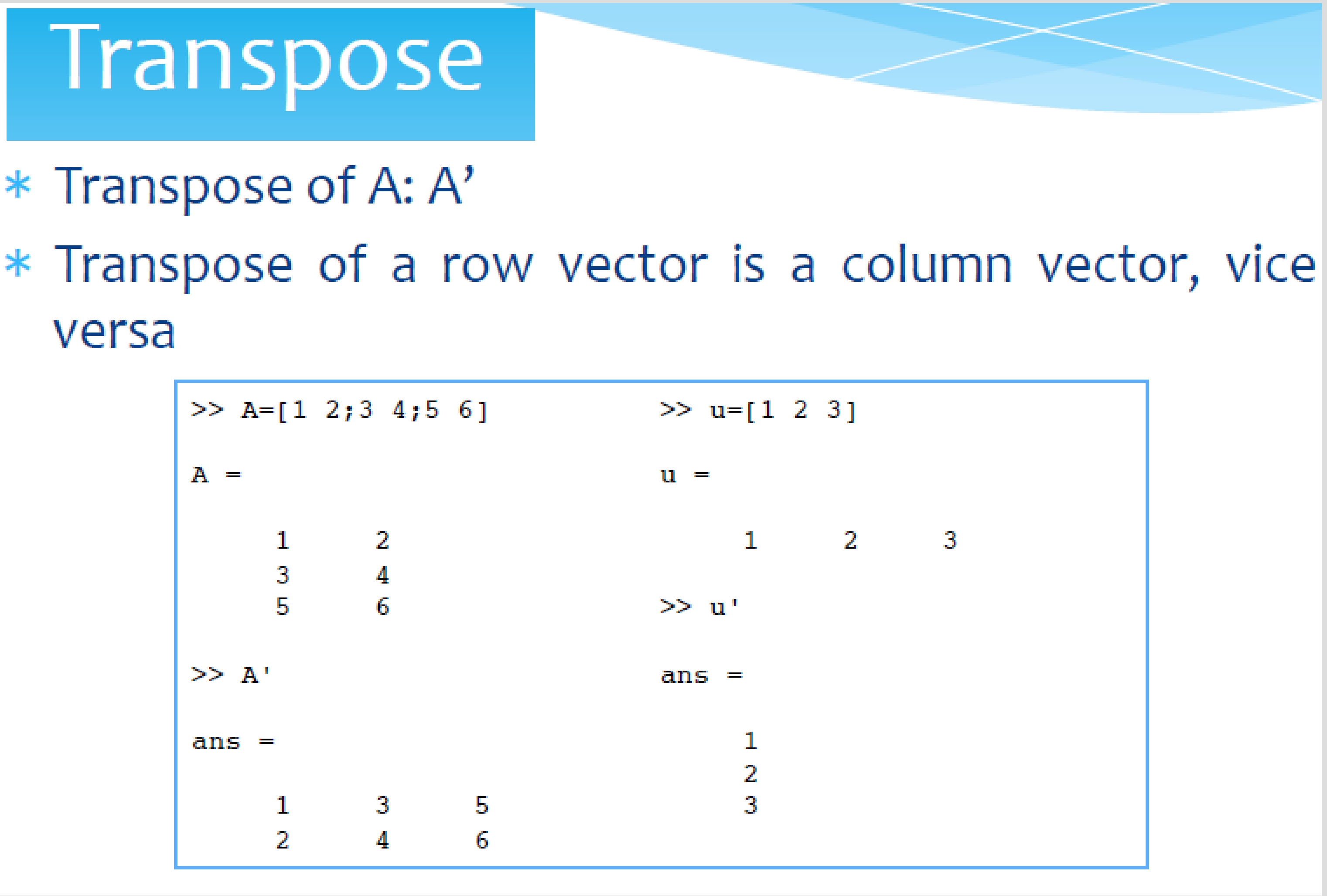
- Use Matlab to verify that AB=BA is not necessarily true
- Observe the difference between A.*B and A*B
- E.g. Use

```
A =
      1     2
      3     4

B =
      4     3
      2     1
```



```
>> A=[1 2;3 4]
A =
     1     2
     3     4

>> B=[4 3;2 1]

B =
     4     3
     2     1
```

```
>> A*B

ans =
     8     5
    20    13

>> B*A

ans =
    13    20
     5     8

>> A.*B

ans =
     4     6
     6     4
```

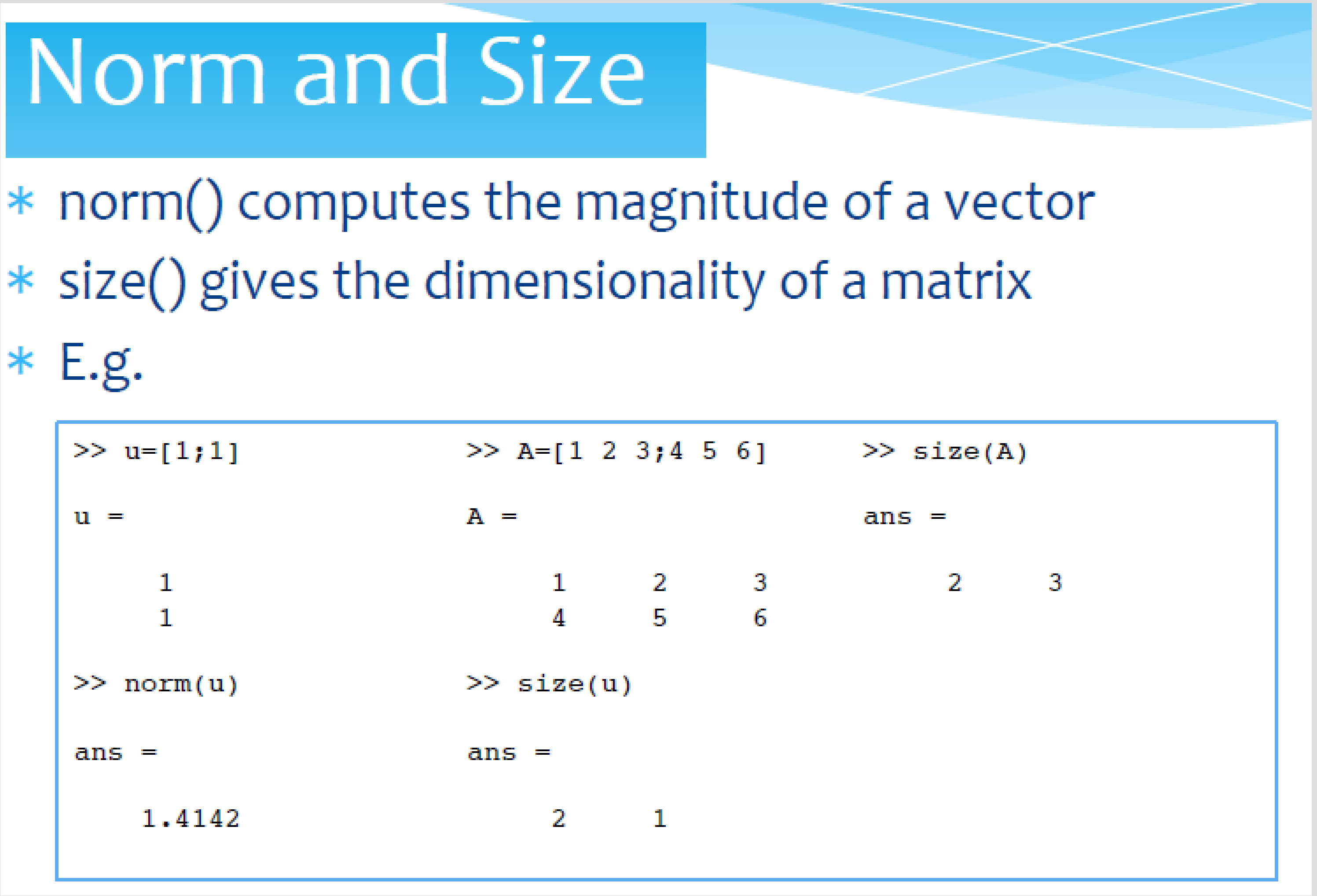


# Transpose

- Transpose of A: A’
- Transpose of a row vector is a column vector, vice versa

```
>> A=[1 2;3 4;5 6]

A =

     1     2
     3     4
     5     6

>> A'

ans =

     1     3     5
     2     4     6
```

```
>> u=[1 2 3]

u =

     1     2     3

>> u'

ans =

     1
     2
     3
```



# Norm and Size

- norm() computes the magnitude of a vector
- size() gives the dimensionality of a matrix
- E.g.

```
>> u=[1;1]

u =

     1
     1

>> norm(u)

ans =

    1.4142
```

```
>> A=[1 2 3;4 5 6]

A =

     1     2     3
     4     5     6

>> size(u)

ans =

     2     1
```

```
>> size(A)

ans =

     2     3
```